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A BOUNDARY ELEMENT METHOD FOR THE INTERACTION OF FREE-SURFACE WAVES WITH A VERY LARGE FLOATING FLEXIBLE PLATFORM

A. J. Hermans
TU Delft, Faculty of Information Technology and Systems,
Department of Applied Mathematical Analysis, Mekelweg 4
2628 CD Delft, The Netherlands

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In this paper a derivation is given of an integro-differential equation for the determination of the deflection of a large floating flexible platform, excited by waves. The formulation is presented for the general three-dimensional case where also a current may be present. For the two-dimensional case (1-D platform) a solution method with orthogonal function expansions turns out to be nonconvergent and therefore we have chosen to use a finite-difference method to tackle the fourth-order derivatives. This method seems to be efficient and may be extended to the general three-dimensional case. In this paper, we present 2-D results only. For the zero current situation the amplitude of the deflection of the platform together with the reflection and transmission coefficients are shown. Furthermore, for several values of the wavelength, the influence of the current is studied. The analysis for the response of a 2-D platform to a moving pressure point is then given.

1. INTRODUCTION

Traditionally, there is much interest in literature in the behaviour of sea ice influenced by waves. Squire *et al.* (1995) have presented an interesting overview of the research carried out in the field of wave–ice interaction. Recently, we became interested in a similar phenomenon, namely the behaviour of large floating flexible platforms. These studies are mainly carried out for the design of large floating airports. The construction of such platforms is under consideration in several parts of the world, so they may meet several wave climates, ranging from sheltered areas to open areas where severe wave conditions may occur. Also a strong tidal current may be present. In this paper, we present an approach to describe the behaviour of a platform of general shape in current and influenced by long crested waves. The method to solve this problem is based on a boundary element method. For the one-dimensional platform with normal incident waves the numerical approach will be explained. A derivation of an expansion in orthogonal functions is described; however, this approach does not lead to reliable results in the range of parameters considered. It is decided to solve the resulting integral–differential equation by means of a finite-difference scheme.

First, we derive a formulation for the general two-dimensional platform. In the cases with or without current we obtain two different integral equations. In our approach, they have the same structure and can be solved in a similar way. The main differences are that the free-surface Green's functions differ and that in the current case the derivative with respect to z instead of the Green's function itself has to be taken into account under the integral.

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A detailed treatment of the two-dimensional case will be presented. This is done because it gives insight into how the method can be treated in the more general three-dimensional problem.

2. MATHEMATICAL FORMULATION

In this section, we derive the general formulation for the diffraction of waves by a flexible platform of general geometric form. We restrict ourselves to platforms with constant elastic properties. This restriction can be weakened later on.

The fluid is incompressible, so we introduce the velocity potential $\bar{\Phi}(\mathbf{x}, t) = \nabla \mathbf{V}(\mathbf{x}, t)$, where $\mathbf{V}(\mathbf{x}, t)$ is the fluid velocity vector. We assume that the potential function can be split up into a steady and an unsteady part, while the steady part is approximated by Ux. Hence, we write

$$\bar{\Phi}(\mathbf{x}, t) = Ux + \Phi(\mathbf{x}, t),$$

and we get for the potential function $\Phi(\mathbf{x}, t)$ the Laplace equation

$$\Delta \Phi = 0 \quad \text{in the fluid} \tag{1}$$

together with the linearized kinematic condition, $\Phi_z = w_t + U w_x$, and dynamic condition, $p/\rho = -\Phi_t - U\Phi_x - gw$, at the linearized free water surface $z = 0$, where $w(x, y, t)$ denotes the free-surface elevation, and ρ is the density of the water. The linearized free-surface condition outside the platform becomes

$$\left\{ \frac{\partial^2}{\partial t^2} + 2U \frac{\partial^2}{\partial x \partial t} + U^2 \frac{\partial^2}{\partial x^2} \right\} \Phi + g \frac{\partial \Phi}{\partial z} = 0 \tag{2}$$

at $z = 0$ and $(x, y) \in \mathscr{F}$.

The platform is assumed to be a thin layer at the free-surface $z = 0$, which seems to be a good model for a shallow draft platform. The platform is modelled as an elastic plate with zero thickness. To describe the deflection w we apply the thin-plate theory, which leads to an equation for w of the form

$$m \frac{\partial^2 w}{\partial t^2} = -D \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)^2 w + p|_{z=0}, \tag{3}$$

where m is the mass of unit area of the platform while D is its equivalent flexural rigidity. We apply the operator $(\partial/\partial t) + U(\partial/\partial x)$ to equation (3) and use the kinematic and dynamic condition to arrive at the following equation for Φ at $z = 0$ and in the platform area $(x, y) \in \mathscr{P}$:

$$\left\{ \frac{D}{\rho g} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)^2 + \frac{m}{\rho g} \frac{\partial^2}{\partial t^2} + 1 \right\} \frac{\partial \Phi}{\partial z} + \frac{1}{g} \left\{ \frac{\partial^2}{\partial t^2} + 2U \frac{\partial^2}{\partial x \partial t} + U^2 \frac{\partial^2}{\partial x^2} \right\} \Phi = 0. \tag{4}$$

The free edges of the platform are free of shear forces and moment. We assume that the radius of curvature, in the horizontal plane, of the edge is large. Hence, the edge may be considered to be straight locally. We then approximate the boundary conditions at the edge by

$$\frac{\partial^2 w}{\partial n^2} + \nu \frac{\partial^2 w}{\partial s^2} = 0 \quad \text{and} \quad \frac{\partial^3 w}{\partial n^3} + (2 - \nu) \frac{\partial^3 w}{\partial n \partial s^2} = 0, \tag{5}$$

where ν is Poisson's ratio, n is in the normal direction in the horizontal plane, along the edge, and s denotes the arc-length along the edge.

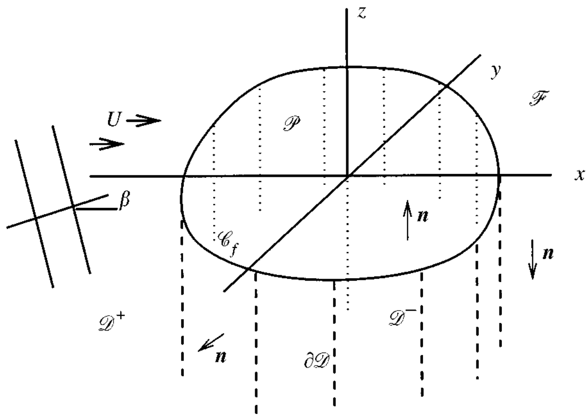

Figure 1. Definition of the geometry.

We consider the diffraction of a plane wave, incident under an angle β with respect to the undisturbed current, U, in the direction of the positive x-axis. The potential function Φ has to be decomposed into a steady and an unsteady part. However, we assume that the steady in-coming current is not influenced by the platform. This is a reasonable assumption for a shallow draft platform, that is kept at on location by means of an anchoring system.

The harmonic wave can be written as $\Phi(\mathbf{x}, t) = \phi(\mathbf{x})\mathrm{e}^{-\mathrm{i}\omega t}$, and the undisturbed incident wave equals

$$\phi^{\mathrm{inc}}(\mathbf{x}) = \frac{\zeta_0}{g\omega_0} \exp\{\mathrm{i}k_0(x\cos\beta + y\sin\beta) + k_0 z\}, \tag{6}$$

where ζ_0 is the wave height, ω the frequency of encounter, while $\omega_0 = \omega - k_0 U\cos\beta$ defines the wave number $k_0 = \omega_0^2/g$ for the infinitely deep-water case.

The fluid domain will be split into two regions. We define the region underneath the platform as $\mathscr{D}^-$ and the region towards infinity as $\mathscr{D}^+$, while the interface is denoted by $\partial\mathscr{D}$. The potential function in $\mathscr{D}^+$ is written as a superposition of the incident wave potential and a diffracted wave potential, as follows:

$$\phi(\mathbf{x}) = \phi^{\mathrm{inc}}(\mathbf{x}) + \phi^+(\mathbf{x}), \tag{7}$$

while the total potential in $\mathscr{D}^-$ is denoted by $\phi^-(\mathbf{x})$, as shown in Figure 1. It will be shown that this choice leads to an interesting way to derive an integral equation, that can be solved numerically. At the dividing surface $\partial\mathscr{D}$ we require continuity of the total potential and its normal derivative.

We introduce the Green's function $\mathscr{G}(\mathbf{x}, \boldsymbol{\xi})$ that fulfills $\Delta\mathscr{G} = 4\pi\delta(\mathbf{x} - \boldsymbol{\xi})$, the free surface and the radiation condition. Green's functions for several free-surface problems can be found in Wehausen & Laitone (1960). An appropriate function is given in the next section for a particular problem. In principle, we can treat cases with or without current. In the next subsection, we derive the integral equations for the zero current case. The formulation for the current case is more complicated and will be treated separately.

2.1. Zero Current–Velocity

We apply Green's theorem to the potentials ϕ^+ and ϕ^-, respectively. This leads to the following approach.

For $\mathbf{x} \in \mathscr{D}^+$, we have

$$4\pi\phi^+ = -\int_{\partial\mathscr{D}\cup\mathscr{F}} \left(\phi^+ \frac{\partial\mathscr{G}}{\partial n} - \mathscr{G}\frac{\partial\phi^+}{\partial n}\right) \mathrm{d}S,$$

$$0 = \int_{\partial\mathscr{D}\cup\mathscr{P}} \left(\phi^- \frac{\partial\mathscr{G}}{\partial n} - \mathscr{G}\frac{\partial\phi^-}{\partial n}\right) \mathrm{d}S. \tag{8}$$

If $\mathbf{x} \in \mathscr{D}^-$, we have

$$0 = -\int_{\partial\mathscr{D}\cup\mathscr{F}} \left(\phi^+ \frac{\partial\mathscr{G}}{\partial n} - \mathscr{G}\frac{\partial\phi^+}{\partial n}\right) \mathrm{d}S,$$

$$4\pi\phi^- = \int_{\partial\mathscr{D}\cup\mathscr{P}} \left(\phi^- \frac{\partial\mathscr{G}}{\partial n} - \mathscr{G}\frac{\partial\phi^-}{\partial n}\right) \mathrm{d}S. \tag{9}$$

The integrals over $\mathscr{F}$ become zero, due to the zero-current free-surface condition for $\mathscr{G}$ and ϕ^+. We add up the two expressions in equation (9) and use the free-surface condition for the Green's function and the potential ϕ^+, which leads to

$$4\pi\phi^- = \int_{\partial\mathscr{D}} \left([\phi]\frac{\partial\mathscr{G}}{\partial n} - \mathscr{G}\left[\frac{\partial\phi}{\partial n}\right]\right) \mathrm{d}S + \int_{\mathscr{P}} \left(\frac{\omega^2}{g}\phi^- - \phi_\zeta^-\right)\mathscr{G}\,\mathrm{d}S \quad \text{for } \mathbf{x} \in \mathscr{D}^-, \tag{10}$$

where we use the notation $[\cdots]$ for the jump of the function concerned. Furthermore, we use the jump condition between the potentials ϕ^+ and ϕ^- and their derivatives. For the total potential the jumps are zero, so we obtain

$$4\pi\phi^- = \int_{\partial\mathscr{D}} \left(\phi^{\mathrm{inc}}\frac{\partial\mathscr{G}}{\partial n} - \mathscr{G}\frac{\partial\phi^{\mathrm{inc}}}{\partial n}\right) \mathrm{d}S - \int_{\mathscr{P}} \left(\frac{m\omega^2}{\rho g}\phi_\zeta^- - \frac{D}{\rho g}\left(\frac{\partial^2}{\partial\xi^2} + \frac{\partial^2}{\partial\eta^2}\right)^2\phi_\zeta^-\right)\mathscr{G}\,\mathrm{d}S, \tag{11}$$

where we have used relation (4) for ϕ^- at the platform. Relation (11) is suitable for further manipulation to end up with a differential-integral equation, that can be solved numerically. The Green's function itself has a weak singularity, so we may take the limit $z \to 0$ and use equation (4) to express ϕ^- in terms of an operator acting on ϕ_z^-. Furthermore, we notice that the first integral on the right-hand side of equation (11) can be simplified significantly. This term is independent of the parameters of the platform, hence it is the same if there is no platform present; it therefore equals $4\pi\phi^{\mathrm{inc}}$. This can also be verified by manipulating the integrals. We arrive at the following equation valid at $z = 0$:

$$\begin{aligned} &4\pi\left(\phi_z^- - \frac{m\omega^2}{\rho g}\phi_z^- + \frac{D}{\rho g}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2\phi_z^-\right) \\ &\quad + \frac{\omega^2}{g}\int_{\mathscr{P}} \left(\frac{m\omega^2}{\rho g}\phi_\zeta^- - \frac{D}{\rho g}\left(\frac{\partial^2}{\partial\xi^2} + \frac{\partial^2}{\partial\eta^2}\right)^2\phi_\zeta^-\right)\mathscr{G}\,\mathrm{d}S \\ &\quad = \frac{\omega^2}{g}\int_{\partial\mathscr{D}} \left(\phi^{\mathrm{inc}}\frac{\partial\mathscr{G}}{\partial n} - \mathscr{G}\frac{\partial\phi^{\mathrm{inc}}}{\partial n}\right) \mathrm{d}S = 4\pi\phi_z^{\mathrm{inc}}. \end{aligned} \tag{12}$$

Equation (12), together with its boundary conditions (5) where we use the relation $-\mathrm{i}\omega w = \phi_z^-$, has to be solved numerically. In the following section, it will be shown how this is done in the two-dimensional case. One must realize that the step from equation (11) towards equation (12) cannot be carried out in the case of nonzero current. However, there is an other way to arrive at the same result. It is clear that one may differentiate eqution (11) with respect to z and take the limit to $z = 0$ afterwards. This operation is not trivial but it leads to equation (12) directly, without making use of the actual free-surface condition anymore. One must be very careful in treating the singularity of $\mathcal{G}_z$ in the case that both z and ζ tend to zero. Details of this step, for a different analysis, are described by Noblesse (1982). It the next subsection it will be shown that in the case of a current this approach has to be followed.

2.2. Nonzero Current Velocity

In the case of a steady current some of the steps carried out above need some special care. It is well known in ship hydrodynamics that application of the free-surface condition in Green's theorem gives rise to an extra line integral along the intersection of the body (in our case the platform) and the free surface; see Brard (1972), Chang (1977) and Hermans & Huijsmans (1987). If we follow the same steps as shown before, we arrive at the following expression for ϕ^-:

$$4\pi\phi^- = \int_{\partial\mathcal{D}} \left(\phi^{\text{inc}} \frac{\partial \mathcal{G}}{\partial n} - \mathcal{G} \frac{\partial \phi^{\text{inc}}}{\partial n} \right) \mathrm{d}S - \frac{2\mathrm{i}U\omega}{g} \int_{\mathscr{C}_f} \phi^{\text{inc}} \mathcal{G} \, \mathrm{d}\zeta$$

$$+ \frac{U^2}{g} \int_{\mathscr{C}_f} \left[\phi^{\text{inc}} \frac{\partial \mathcal{G}}{\partial \xi} - \left(\alpha_s \frac{\partial \phi^{\text{inc}}}{\partial s} - \alpha_n \frac{\partial \phi^{\text{inc}}}{\partial n} \right) \mathcal{G} \right] \mathrm{d}\zeta$$

$$- \int_{\mathscr{P}} \left[\frac{m\omega^2}{\rho g} \phi_\zeta^- - \frac{D}{\rho g} \left(\frac{\partial^2}{\partial \xi^2} + \frac{\partial^2}{\partial \eta^2} \right)^2 \phi_\zeta^- \right] \mathcal{G} \, \mathrm{d}S, \tag{13}$$

where we introduce α_s and α_n as the cosine of the tangent and the normal vector to the x-axis, respectively. At this stage the difference from the zero current case is only an extra known integral along the water-line. Again, this expression can be simplified as the first three integrals equal $4\pi\phi^{\text{inc}}$. The next step in arriving at an integral equation for the unknown ϕ_z^- at $z = 0$ can only be done by differentiation of equation (13) with respect to z, and taking the limit to $z = 0$ afterwards.

We shall repeat the theory of section 9 of Noblesse (1982) for the forward speed case. The Green's function $\mathcal{G}(\mathbf{x}, \boldsymbol{\xi})$ can be found in Wehausen & Laitone (1960). The general form is

$$\mathcal{G}(\mathbf{x}, \boldsymbol{\xi}) = -\frac{1}{r} + \frac{1}{r_1} + \psi^+(\mathbf{x}, \boldsymbol{\xi}), \tag{14}$$

where $r = [R^2 + (z - \zeta)^2]^{1/2}$, $r_1 = [R^2 + (z + \zeta)^2]^{1/2}$, $R = [(x - \xi)^2 + (y - \eta)^2]^{1/2}$, and $\psi^+(\mathbf{x}, \boldsymbol{\xi})$ is defined as follows:

$$\psi^+(\mathbf{x}, \boldsymbol{\xi}) = -\frac{2g}{\pi} \left(\int_0^\gamma \int_0^\infty + \int_\gamma^{\pi/2} \int_{\mathscr{L}_1} + \int_{\pi/2}^{\pi} \int_{\mathscr{L}_2} \right) F(\theta, k) \, \mathrm{d}\theta \, \mathrm{d}k.$$

Here

$$F(\theta, k) = \frac{k \mathrm{e}^{k[z + \zeta + \mathrm{i}(x - \xi)\cos\theta]} \cos[k(y - \eta)\sin\theta]}{gk - (\omega + kU\cos\theta)^2}$$

with

$$\gamma = \begin{cases} 0 & \text{if } \tau < \frac{1}{4} \\ \arccos \frac{1}{4\tau} & \text{if } \tau \geq 0, \end{cases}$$

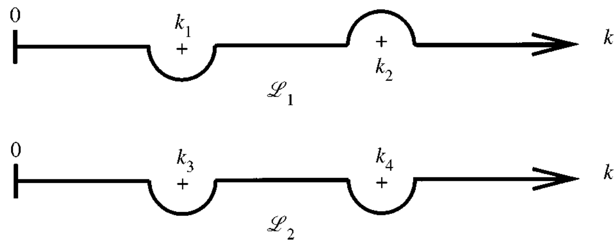

Figure 2. Contours of integration.

and $\tau = U\omega/g$. The contours have to be chosen such that the radiation conditions are fulfilled. They are given in Figure 2, where the poles of the integrand are given by

$$\sqrt{gk_1}, \sqrt{gk_3} = \frac{1 - \sqrt{1 - 4\tau\cos\theta}}{2\tau\cos\theta}\,\omega,$$

$$\sqrt{gk_2}, -\sqrt{gk_4} = \frac{1 + \sqrt{1 - 4\tau\cos\theta}}{2\tau\cos\theta}\,\omega.$$

We now make use of an alternative way to write the Green's function $\mathscr{G}(\mathbf{x}, \boldsymbol{\xi})$. If one notices that the function $\psi^+(\mathbf{x}, \boldsymbol{\xi})$ depends on $z + \zeta$ and not on $z - \zeta$ it may be written as

$$\mathscr{G}(\mathbf{x}, \boldsymbol{\xi}) = -\frac{1}{r} - \frac{1}{r_1} + \psi^-(\mathbf{x}, \boldsymbol{\xi}), \tag{15}$$

where $\psi^-(\mathbf{x}, \boldsymbol{\xi})$ is defined accordingly. If we now look at the free-surface condition (2) we rewrite it for $\mathscr{G}(\mathbf{x}, \boldsymbol{\xi})$ as follows

$$g\mathscr{G}_z = -\mathscr{D}[\mathscr{G}] - g(1/r + 1/r_1)_z + g\psi_z^-(\mathbf{x}, \boldsymbol{\xi}) + \mathscr{D}[-1/r + 1/r_1] + \mathscr{D}[\psi^+(\mathbf{x}, \boldsymbol{\xi})], \tag{16}$$

where the operator $\mathscr{D}[\ldots]$ is defined as

$$\mathscr{D}[\ldots] = \left(-\omega^2 - 2\mathrm{i}\omega U\frac{\partial}{\partial x} + U^2\frac{\partial^2}{\partial x^2}\right)[\cdots]. \tag{17}$$

The free-surface condition shows that we have $g\mathscr{G}_z + \mathscr{D}[\mathscr{G}] = 0$ on $z = 0$ if $\zeta < 0$. It is also obvious that $\mathscr{D}[-1/r + 1/r_1] = 0$ on $z = 0$ and $(1/r + 1/r_1)_z = 0$ on $z = 0$ if $\zeta < 0$. Just as in Noblesse's case one may conclude that

$$g\psi_z^-(\mathbf{x}, \boldsymbol{\xi}) + \mathscr{D}[\psi^+(\mathbf{x}, \boldsymbol{\xi})] = 0 \quad \text{for all } z \leq 0. \tag{18}$$

Hence, one may write

$$\mathscr{G}_z = -(1/r + 1/r_1)_z - 1/g\mathscr{D}[\mathscr{G} + 1/r - 1/r_1], \tag{19}$$

while $\mathscr{G}$ may be written as $\mathscr{G} = -1/r + g(R, z + \zeta)$. So, if one differentiates (14) with respect to z and takes the limit $z \to 0$, one obtains the following Fredholm integral equation:

$$4\pi\left(\phi_z^- - \frac{m\omega^2}{\rho g}\phi_z^- + \frac{D}{\rho g}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2\phi_z^-\right)$$

$$= 4\pi\phi_z^{\mathrm{inc}} - \int_{\mathscr{P}}\left(\frac{m\omega^2}{\rho g}\phi_\zeta^- - \frac{D}{\rho g}\left(\frac{\partial^2}{\partial \xi^2} + \frac{\partial^2}{\partial \eta^2}\right)^2\phi_\zeta^-\right)\mathscr{G}_z\,\mathrm{d}S. \tag{20}$$

If we replace the function ϕ_z^- by $-\mathrm{i}\omega w + U w_x$, we obtain an equation for the elevation of the platform. The extra derivative with respect to x has to be treated with upwind differencing. Again we have the boundary conditions (5) for the function w at the free edge of the platform. In the next section, we present details of the numerical approach to solving the integro-differential equation.

3. ONE-DIMENSIONAL PLATFORM

3.1. No Current, Perpendicular Angle of Incidence

A good demonstration of the method is the treatment of the two-dimensional problem. First, we give an outline of the approach for the situation that no current is present and the incident waves are normal to the edge. The integro-differential equation for the potential ϕ^- at $z = 0$ becomes

$$-\left(\kappa\phi_z^- - \mathscr{D}\frac{\partial^4\phi_z^-}{\partial x^4}\right) + \delta^2\int_{\mathscr{P}}\left(\mu\phi_\zeta^- - \mathscr{D}\frac{\partial^4\phi_\zeta^-}{\partial\xi^4}\right)\mathscr{G}\,\mathrm{d}\xi = \phi_z^{\mathrm{inc}}, \tag{21}$$

where we have introduced the parameters

$$\mu = \frac{m\omega^2}{\rho g},\quad \kappa = \mu - 1,\quad \delta^2 = \frac{\omega^2}{2\pi g} = \frac{k_0}{2\pi}\quad \text{and}\quad \mathscr{D} = \frac{D}{\rho g}$$

together with the boundary conditions

$$\frac{\partial^2\phi_z^-}{\partial x^2} = \frac{\partial^3\phi_z^-}{\partial x^3} = 0 \quad \text{at } x = 0 \text{ and } x = l,$$

l being the length of the plate. The Green's function, obeying the radiation condition, has the form

$$\begin{aligned}\mathscr{G}(\mathbf{x},\boldsymbol{\xi}) - \ln r &= -\ln r_1 - 2\int_{\mathscr{L}'}\frac{1}{k-k_0}\,\mathrm{e}^{k(z+\zeta)}\cos k(x-\xi)\,\mathrm{d}k\\ &= \int_{\mathscr{L}'}\left(\frac{k+k_0}{(k-k_0)k}\,\mathrm{e}^{k(z+\zeta)}\cos k(x-\xi) + \frac{1}{k}\,\mathrm{e}^{-k}\right)\mathrm{d}k,\end{aligned} \tag{22}$$

where the contour $\mathscr{L}'$ is in the complex k-plane from $k = 0$ to ∞ that passes, due to the radiation condition, underneath the pole of the integrand $k = k_0 = \omega^2/g$, see Figure 3.

To solve equation (21) it is possible to make use of an expansion in eigenmodes for the deflection of the plate, hence in terms of eigenfunctions of

$$\frac{\partial^4\psi}{\partial x^4} = \lambda^4\psi \quad \text{with} \quad \frac{\partial^3\psi}{\partial x^3} = \frac{\partial^2\psi}{\partial x^2} = 0 \tag{23}$$

at $x = 0$ and l. The orthonormal eigenfunctions are $\psi_n(x) = \bar{\psi}_n(x)/[\int_0^l \bar{\psi}_n^2(x)\,\mathrm{d}x]^{1/2}$, defined by

$$\bar{\psi}_n(x) = (\cos\lambda_n x + \cosh\lambda_n x) - \frac{(\cos\lambda_n l - \cosh\lambda_n l)}{(\sin\lambda_n l - \sinh\lambda_n l)}(\sin\lambda_n x + \sinh\lambda_n x), \tag{24}$$

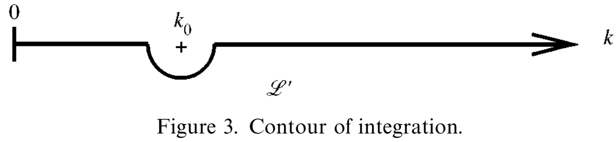

Figure 3. Contour of integration.

where λ_n is the eigenvalue of the problem, obtained from the zeros of

$$\cos\lambda_n l \cosh\lambda_n l = 1 \quad \text{with} \quad n = 1, 2, \ldots.$$

Associated with the double root $\lambda_0 = 0$ we have the orthogonal eigenfunctions

$$\psi_{-1} = 1 + \sqrt{3}x/l \quad \text{and} \quad \psi_0 = 1 - \sqrt{3}x/l.$$

We expand the deflection of the platform $w(x,t) = \tilde{w}(x)\mathrm{e}^{-\mathrm{i}\omega t}$ in terms of the eigenfunctions, just defined. The function ϕ_z^- is therefore written as

$$\phi_z^-(x,0) = -\mathrm{i}\omega\tilde{w}(x) = -\mathrm{i}\omega \sum_{n=-1}^{N} a_n\psi_n(x).$$

If we multiply equation (20) with $\psi_m(x)$ and integrate with respect to x, we get the following set of equations for the coefficients a_m:

$$(\mathscr{D}\lambda_m^4 - \kappa)a_m - \delta^2 \sum_{n=-1}^{N} a_n(\mathscr{D}\lambda_n^4 - \mu)g_{m,n} = \frac{\mathrm{i}}{\omega}c_m \tag{25}$$

for $m = -1, 0, \ldots, N$, where we have introduced

$$g_{m,n} = \int_0^l \int_0^l \psi_n(\xi)\mathscr{G}(x,0,\xi,0)\psi_m(x)\,\mathrm{d}x\,\mathrm{d}\xi \tag{26}$$

and

$$c_m = \int_0^l \phi_z^{\mathrm{inc}}(x,0)\psi_m(x)\,\mathrm{d}x. \tag{27}$$

The integration with respect to x and ξ in equation (26) and with respect to x in equation (27) is carried out analytically, while the integration with respect to k in equation (22) is carried out numerically. This last step can be done by integrating along the contour $\mathscr{L}'$ as given in Figure (3) or by performing a contour deformation first.

This method is applied for a platform with a length $l = 300$ m in long waves and with several values of the flexural rigidity. In Figure 4 we show computations of the amplitude of the deflection for $\lambda/l = 0{\cdot}5$, $0{\cdot}1$, respectively, and $D/\rho g = 1 \times 10^7$ m^4, computed with the

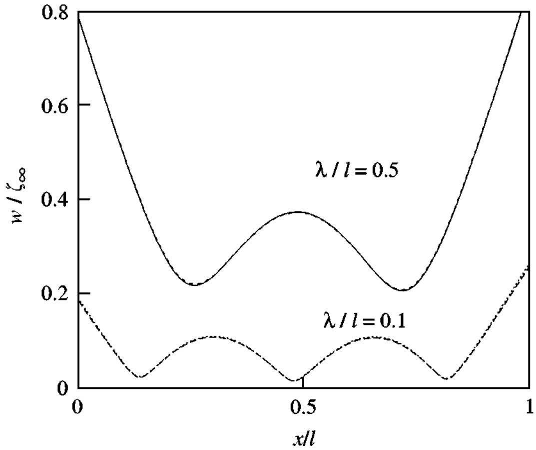

Figure 4. Amplitude of the deflection for $D/\rho g = 10^7$ m^4, $m/\rho = 1/4$ m and $l = 300$ m, computed by the two methods.

method described above and compared with a finite difference approach. The results compare very well. No difference between the results is visible. A disadvantage of the expansion method is the fact that this approach cannot be used in the case of a general two-dimensional platform, due to the boundary conditions at the edge of the platform.

We have chosen an approach that can be extended to general geometric forms and where one may take care of the local behaviour easily. In this case, we employ a finite-difference approach with equidistant grid points, sufficiently fine to cover the local effects. In the two-dimensional case it is advisable to use a nonuniform grid. The fourth-order derivative is represented by a five-point central difference scheme. This is also used for the end-points of the platform, by introduction of two grid points outside the physical plane. The two boundary conditions at the boundaries of the platform give rise to four extra equations. We therefore obtain a square $(n + 4, n + 4)$ matrix equation if we consider n collocation point plus four external points. The matrix is fully populated due to the integral, and to compute its coefficients the integration with respect to ξ is carried out analytically, before the integration with respect to k in the definition of the Green's function. For the integration with respect to k, the contour in the complex k plane is deformed, leading to a residue from the pole plus an integration along the imaginary axis. This last integration is carried out with a fast Gauss integration procedure for exponential integrals.

Examples of the computation of the amplitude of the deflection w of the platform are shown in Figure 5, while in Figure 6 the reflection and transmission coefficients are given.

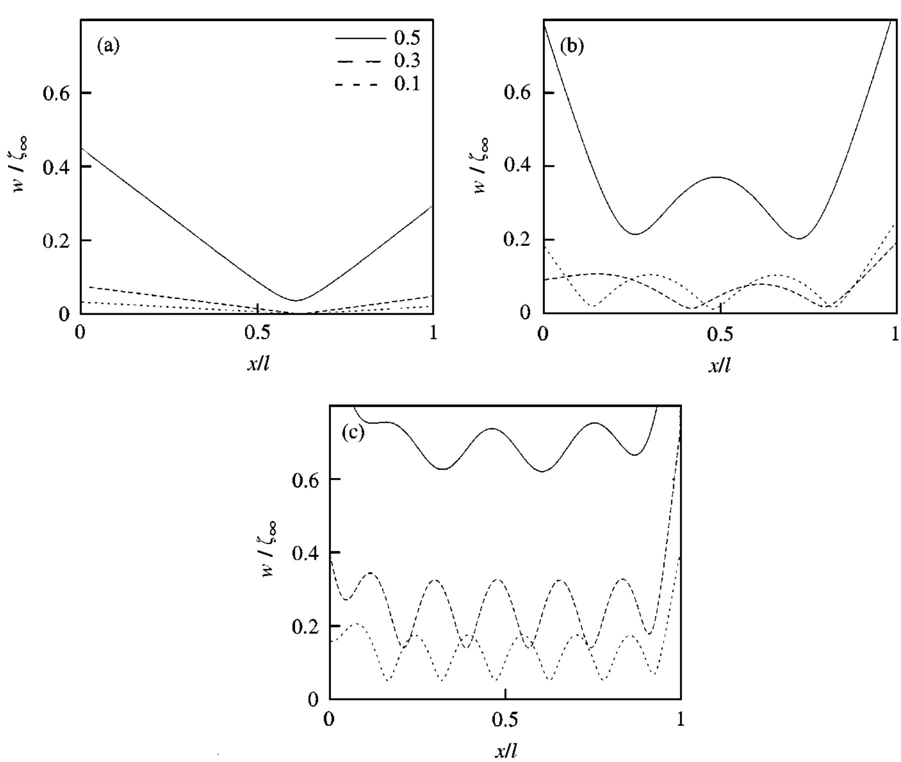

Figure 5. Amplitude of the deflection for a platform with $l = 300$ m, $\lambda/l = 0{\cdot}5$, $0{\cdot}3$, $0{\cdot}1$ and several values of the rigidity: (a) $D/\rho g = 10^{10}$ m^4; (b) $D/\rho g = 10^7$ m^4; (c) $D/\rho g = 10^5$ m^4.

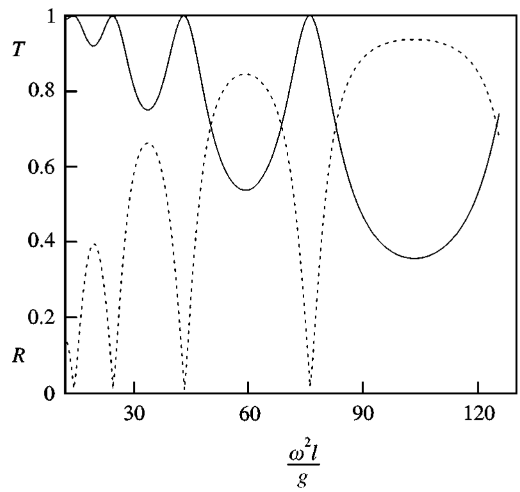

Figure 6. Reflection and transmission coefficients with $D/\rho g = 10^5$ m^4, $m/\rho = 1/4$ m and $l = 300$ m.

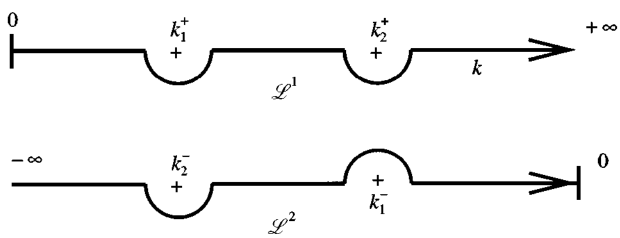

Figure 7. Contours of integration.

3.2. With a Current, Perpendicular Angle of Incidence

The integro-differential equation now becomes

$$-\left(\kappa\phi_z^- - \frac{D}{\rho g}\frac{\partial^4\phi_z^-}{\partial x^4}\right) + \frac{1}{2\pi}\int_{\mathscr{P}}\left(\mu\phi_\zeta^- - \frac{D}{\rho g}\frac{\partial^4\phi_\zeta^-}{\partial \xi^4}\right)\tilde{\mathscr{G}}_z\,\mathrm{d}\xi = \phi_z^{\mathrm{inc}}. \tag{28}$$

The Green's function has the form

$$\tilde{\mathscr{G}}(\mathbf{x},\boldsymbol{\xi}) = \ln\left(\frac{r}{r_1}\right) + 2\int_{\mathscr{L}^1}\frac{1}{\omega^2/g - (2\tau+1)k + k^2(U^2/g)}\,\mathrm{e}^{\mathrm{i}k(x-\xi)+k(z+\zeta)}\,\mathrm{d}k$$
$$+\,2\int_{\mathscr{L}^2}\frac{1}{\omega^2/g - (2\tau-1)k + k^2(U^2/g)}\,\mathrm{e}^{\mathrm{i}k(x-\xi)-k(z+\zeta)}\,\mathrm{d}k, \tag{29}$$

where $\tau = \omega U/g$ and the contours $\mathscr{L}^1$ and $\mathscr{L}^2$ are defined in Figure 7, in the case $\tau < \frac{1}{4}$. The poles are defined as

$$k_{1,2}^+ = \frac{(2\tau+1) \pm \sqrt{1+4\tau}}{2U^2/g},$$

$$k_{1,2}^- = \frac{(2\tau-1) \pm \sqrt{1-4\tau}}{2U^2/g}.$$

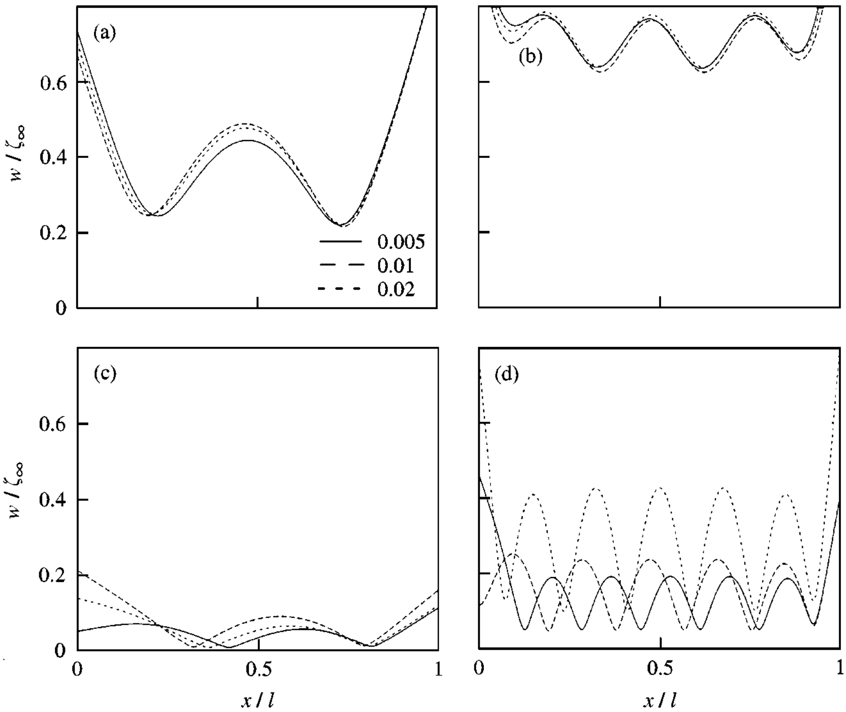

Figure 8. Amplitude of the deflection for $\lambda/l = 0{\cdot}5$, $0{\cdot}1$ and $\tau = 0{\cdot}005$, $0{\cdot}01$, $0{\cdot}02$: (a) $D/\rho g = 10^7$ m^4, $\lambda/l = 0{\cdot}5$; (b) $D/\rho g = 10^5$ m^4, $\lambda/l = 0{\cdot}5$; (c) $D/\rho g = 10^7$ m^4, $\lambda/l = 0{\cdot}1$; (d) $D/\rho g = 10^5$ m^4, $\lambda/l = 0{\cdot}1$.

To obtain an equation for the deflection w we introduce $\phi_z = -\mathrm{i}\omega w + U w_x$. The differential operator in the equation for w becomes of fifth order. The extra derivative can be taken care of by means of upwind differencing, which alters the matrix considerably, compared with the zero speed case. Numerical excercises show that the results converge to those obtained in Figure 5 for $U \to 0$.

For small values of τ the contribution of the poles at $k_2^{+,-}$ becomes highly oscillatory. In the computations, their influence can be taken care of by choosing extremely small grid sizes. It can be shown that their final influence is negligible. For two values of the wavelength, $\lambda/l = 0{\cdot}5$, $0{\cdot}1$, respectively, the computations of the deflection of the platform are shown in Figure 8. The effect of the current is clearly visible.

4. MOVING PRESSURE POINT

In this section, we consider the response of the platform to a moving point source. We consider the effect of an accelerating aircraft before take off or a decelerating one after landing. The process we describe is unstationary. The position of the pressure disturbance is along the x-axis at the position $x = a(t)$. The condition at the platform surface for $t > 0$ becomes

$$\left\{\frac{D}{\rho g}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 + \frac{m}{\rho g}\frac{\partial^2}{\partial t^2} + 1\right\}\frac{\partial \Phi}{\partial z} + \frac{1}{g}\frac{\partial^2 \Phi}{\partial t^2} = -\frac{1}{\rho g}\frac{\partial}{\partial t}\delta(x - a(t))\delta(y) \tag{30}$$

at $z=0$ together with the Laplace equation. We apply a Fourier transform with respect to x and a cosine transform with respect to y:

$$\bar{\Phi}(\alpha,\beta,z,t)=\int_{\infty}^{\infty}\int_{0}^{\infty}\mathrm{e}^{\mathrm{i}\alpha x}\cos\beta y\;\Phi(x,y,z,t)\,\mathrm{d}x\,\mathrm{d}y. \tag{31}$$

This leads to a surface condition of the form

$$\left\{\frac{D}{\rho g}(\alpha^2+\beta^2)^2+\frac{m}{\rho g}\frac{\partial^2}{\partial t^2}+1\right\}\frac{\partial\bar{\Phi}}{\partial z}+\frac{1}{g}\frac{\partial^2\bar{\Phi}}{\partial t^2}=-\frac{1}{\rho g}\frac{\partial}{\partial t}\mathrm{e}^{\mathrm{i}\alpha a(t)}:=-\frac{1}{\rho g}\frac{\partial}{\partial t}f(t) \tag{32}$$

at $z=0$ together with

$$-(\alpha^2+\beta^2)\bar{\Phi}+\bar{\Phi}_{zz}=0\quad\text{for } z<0. \tag{33}$$

We assume that for $t\le 0$ the whole system is at rest, and we apply the Laplace transform with respect to t:

$$\phi(\alpha,\beta,z,s)=\int_0^{\infty}\bar{\Phi}(\alpha,\beta,z,t)\mathrm{e}^{-st}\,\mathrm{d}t. \tag{34}$$

The resulting ordinary differential equation for $\phi(\alpha,\beta,z,s)$ is solved, making use of the kinematic condition $\Phi_z=w_t$ at $z=0$. Finally, the following transform $W(\alpha,\beta,s)$ of the elevation of the platform is obtained:

$$W(\alpha,\beta,s)=\frac{k\mathscr{F}(s)}{(s^2+K^2(k))(mk+\rho)}, \tag{35}$$

where $\mathscr{F}(s)$ is the Laplace transform of $f(t)$, $k^2:=\alpha^2+\beta^2$ and

$$K^2(k):=\frac{Dk^5+k\rho g}{mk+\rho}.$$

The inverse Laplace transform is written as a convolution integral. Introducing $\alpha=k\cos\theta$, $\beta=k\sin\theta$ and carrying out the integration with respect to θ, we obtain

$$w(x,y,t)=-\frac{1}{2\pi}\int_{k=0}^{\infty}\int_{\tau=0}^{t}k\mathscr{H}(k)\sin(K(k)(t-\tau))\times\mathrm{J}_0\left(k\sqrt{(x-a(\tau))^2+y^2}\right)\mathrm{d}k\,\mathrm{d}\tau, \tag{36}$$

where

$$\mathscr{H}(k):=\frac{k}{\{(Dk^5+\rho gk)(mk+\rho)\}^{1/2}}.$$

In the case $D=0$ and $m=0$ we have obtained the classical free-surface result as can be found in Wehausen & Laitone (1960). To compute integral (36) the path of integration in the complex k-plane has to be chosen such that the integrand behaves as closely as possible to a function with a negative exponential, where we write for the zero-order Bessel function J_0 its asymptotic expression for large values of its argument. Some analytic contour deformations can be carried out for the case in which we neglect the effect of the density of the water on the phase of the exponential function, $K(k)=\sqrt{D/m}\,k^2:=\nu k^2$. The integral can be computed directly by an accurate Gauss integration, which is straightforward but rather time-consuming. Fast computer codes can be designed if necessary.

As an example some computations are carried out first for a decelerating unit point source. The results are expressed in dimensionless form based on dividing length by a characteristic dimension L. Thus time is non-dimensionalised by $L^2\sqrt{m/D}$ and force by

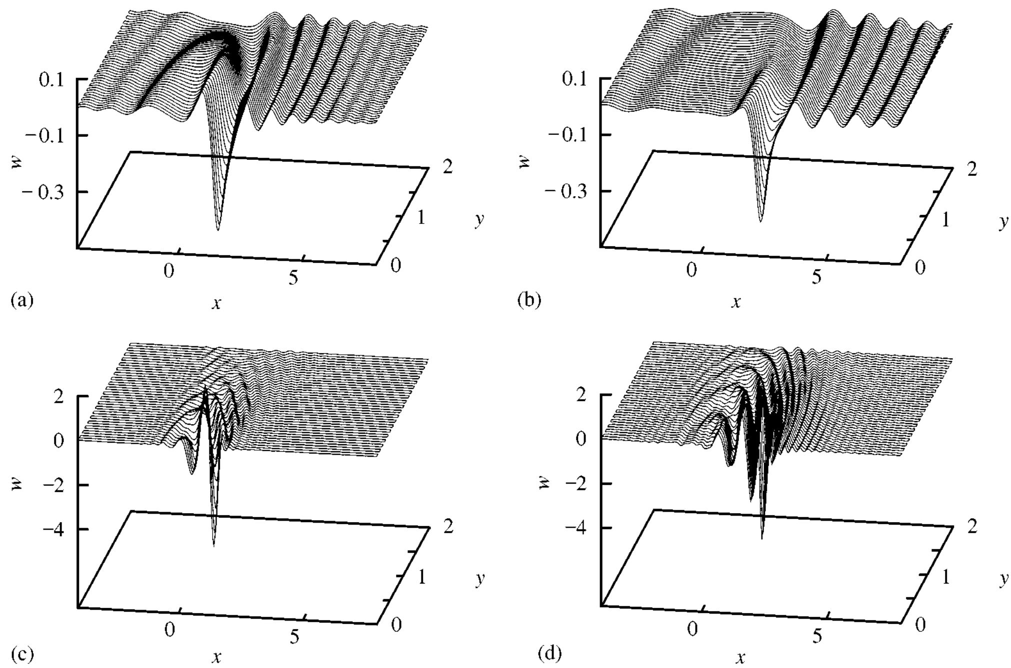

Figure 9. Waveheight for a decelerating unit point source starting at $t = 0$: (a) $DL^4/\rho g = 10^{-3}$, $t = 2$; (b) $DL^4/\rho g = 10^{-3}$, $t = 4$; (c) $DL^4/\rho g = 10^{-5}$, $t = 2$; (d) $DL^4/\rho g = 10^{-5}$, $t = 4$.

D/L. The point source starts at $t = 0$ in the origin and moves along the x-axis with $a(t) = ut - 0{\cdot}1t^2$. Results are given for the case $uL\sqrt{m/D} = 1$ and $m/\rho L = 1/4$. In (a) and (b) $DL^4/\rho g = 10^{-3}$, and in (c) and (d) $DL^4/\rho g = 10^{-5}$. If realistic values for a specific construction are available, the results can be computed. In the figures we see a combination of effects. In front of the point source we see the effect of an impulsive start combined with the moving pattern. It seems that the velocity of the radial wave component is larger than the speed of the point-source.

If we compare Figure 9(a) with Figure 10 we can see the effect of the velocity on the waveheight for a decelerating unit point source.

The effect of the impulsive taking-off becomes apparent if one looks at the effect of a starting pressure point. In Figure 11 we show the surface elevation, at $t = 2$ and 4, of an accelerating point source starting at zero speed in the origin. The position of the point source is given by $a(t) = 0{\cdot}1t^2$. The field is a combination of a steady part and an unsteady part described by an initial-value problem. The initial condition is given by the steady solution. The steady part of a unit point source in the origin has the form

$$w(x, y) = -\frac{1}{2\pi}\int_{k=0}^{\infty} \frac{k}{Dk^4 + \rho g}\,\mathrm{J}_0(kR)\,\mathrm{d}k. \tag{37}$$

For the chosen parameters it can be seen that the circular wave pattern is not present. It is noticed that gravity does not give a substantial contribution in the form of a Kelvin-like pattern. The pattern is dominated by elastic effects.

5. CONCLUSIONS

Results have been presented based on an analysis of a one-dimensional platform in waves and a current. The maximum deflection is shown not to be strongly dependent on the

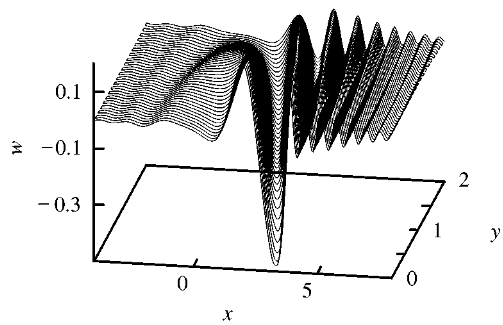

Figure 10. Waveheight for a decelerating unit point source at $t = 2$, starting at $t = 0$, with $u = 2$, $D/\rho g = 10^{-3}$ and $m = \frac{1}{4}$.

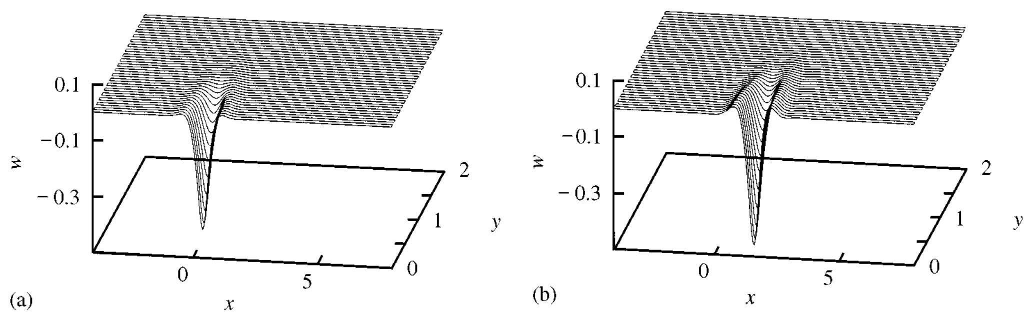

Figure 11. Waveheight for an accelerating unit point source starting at $t = 0$, with $DL^4/\rho g = 10^{-3}$ and $m/\rho L = \frac{1}{4}$: (a) $t = 2$; (b) $t = 4$.

relative stiffness of the platform. The effect of current becomes more pronounced as the relative stiffness is decreased.

Analysis of the response to a moving point force has also been given. In the case of a decelerating force, the results show a radial wave propagating at a higher speed than that of the point force. For an accelerating force, however, under the conditions examined, the radial wave is not present. The wave pattern in the plate is then dominated by elastic effects.

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